

Unit Dimensional Analysis Activity – Version 2

Why? In this activity we will see that it is possible to look at a situation from several points of view, or to take measurements of that same situation using different units of measure. Every measurement has 2 components: **magnitude** and **dimension**. **Magnitude** is the value of the number in the measurement and **dimension** is the unit of measure (e. g. grams, centimeters, inches or liters.)

- *If a measurement is given, can we convert that measurement to different units to meet our needs?*

Model: Car Trip

Given:

90 miles

75 minutes

3 gallons of gasoline

\$12.00

1 bathroom break

27 songs on your iPod®



Group Instructions: When addressing each question, one group member should be assigned the task of reading the question aloud for the rest of the group. The manager should rotate that role among group members throughout the assignment.

Critical Questions:

1. How long does it take to drive 90 miles?
2. How long does it take to drive 180 miles?
3. How many miles can you drive on 3 gallons of gas?
4. How many miles can you drive on 1 gallon of gas?
5. Show how you solved question # 4. Be sure to include the units in your calculations.
6. Show the miles per gallon as a **fraction** (ratio) with numerator and denominator. Which is the numerator? Which is the denominator?
7. Using a grammatically correct sentence describe how you made the choice for # 6.
8. Is there another way to write the **fractional relationship** of gallons and miles? Show this way.
9. Why might you want to write the ratio this 2nd way?

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10. Here are 3 other ratio relationships that we can obtain from the model:

$\frac{1 \text{ bathroom break}}{90 \text{ miles}}$

$\frac{3 \text{ gallons}}{75 \text{ minutes}}$

$\frac{27 \text{ songs}}{\$12.00}$

Write 4 other such relationships that you can obtain from the model:

These relationships are called **Conversion Factors**.

What are the components of a conversion factor?

Using complete sentences consult with your group and come up with a description of a conversion factor. What are its essential components and what is its purpose?

11. Which one of the conversion factors from #10 would you use to determine how long it would take to burn 8 gallons of gas?
12. Construct the conversion factor needed to determine how many songs you would hear in 500 miles.
13. Solve # 12 mathematically. Show your work below and be sure to include units.

Reflections:

14. As a group, write grammatically correct English sentences to describe the objective of the activity at this point. Be prepared to share your answer with the class.
15. After having shared with the class, does your group still agree with your initial assessment of what the objective is?
16. As a group, can you think of a situation when a scientist or chemist might need to use conversion factors to solve a problem? Give an example.

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Exercises:

Using **conversion factors** to solve a problem is called **Dimensional Analysis**. You should now be able to solve the following problems.

17. Solve this problem *without using a calculator*:

$$\frac{6 \times 17 \times 3 \times 13}{13 \times 9 \times 17} =$$

18. Write a mathematical rule that makes this problem easier to solve.

19. Solve this problem:

$$\frac{\text{miles} \times \text{songs} \times \text{gallons}}{\text{miles} \times \text{gallons}} =$$

It is often convenient to represent calculations of this type as a “cancellation line.” The cancellation line for Problem 17 would look like this:

$$\begin{array}{c|c|c|c} 6 & 17 & 3 & 13 \\ \hline & 13 & 9 & 17 \end{array} = 2$$

Cancellation lines can also be used with units:

$$\begin{array}{c|c|c} 5 \text{ days} & 24 \text{ hours} & 60 \text{ minutes} \\ \hline & 1 \text{ day} & 1 \text{ hour} \end{array} = 7200 \text{ min.}$$

Use a cancellation line to solve the remaining problems.

20. How many miles would you have to drive to hear 43 songs? Show how you solve the problem using units and conversion factors.
21. Using your answer from # 20, how many minutes would this take? Again show how you solve the problem using units and conversion factors.
22. Show how you can combine problems # 20 and # 21 into one. Draw a line through any units that cancel. Put your answer on the board.
23. Write a grammatically correct English sentence to describe which unit you will be left with in the answer.

On your own

24. The average human heart beats 72 beats/minute. If you live to be 80 years old, how many times does your heart beat. What conversion factors do you need to know to solve this problem. List these conversion factors.
25. What units should the answer be in? What value would you use to begin the problem and why? Solve the problem and show your work. Include all units and show cancellations of the units.

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Teacher Resources

Prior knowledge needed for this activity:

- Need to know basic units of English measure.
- Need also to know basic algebra involving numerator/denominator cancellations and ratios.

Thoughts on presentation of the activity:

- You might want to model the fractional problem setup first shown in #13 below.

Target Responses for the tasks:

Model: Car Trip

Given:

90 miles

75 minutes

3 gallons of gasoline

\$12.00

1 bathroom break

27 songs on your iPod®



Group Instructions: When addressing each question, one group member should be assigned the task of reading the question aloud for the rest of the group. The manager should rotate that role among group members throughout the assignment.

Critical Questions:

1. How long does it take to drive 90 miles? **75 minutes**
2. How long does it take to drive 180 miles? **150 minutes (2.5 hours)**
3. How many miles can you drive on 3 gallons of gas? **90 miles**
4. How many miles can you drive on 1 gallon of gas? **30 miles**
5. Show how you solved question # 4. Be sure to include the units in your calculations.
There are various ways to show the solution. Students will probably use a ratio.

$$\frac{3 \text{ gal}}{90 \text{ mi}} = \frac{1 \text{ gal}}{x} \quad x = 30 \text{ mi.}$$
6. Show the miles per gallon as a fraction (ratio). Which is the numerator? Which is the denominator? $\frac{90 \text{ mi}}{3 \text{ gal}} = \frac{30 \text{ mi}}{1 \text{ gal}}$
7. Using a grammatically correct sentence describe how you made the choice for # 6.
Miles PER gallon implies miles DIVIDED BY gallons. Mpg is a common ratio and statistic given for cars.
8. Is there another way to write the ratio of gallons and miles? Show this way.
1gal/30 mi
9. Why might you want to write the ratio this 2nd way?
If one wanted to calculate how many gallons of gas it would take to drive a certain distance.

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10. Here are 3 other ratio relationships that we can obtain from the model:

$$\frac{1 \text{ bathroom break}}{90 \text{ miles}}$$

$$\frac{3 \text{ gallons}}{75 \text{ minutes}}$$

$$\frac{27 \text{ songs}}{\$12.00}$$

Write 4 other such relationships that you can obtain from the model:

Variety of responses using any 2 values from the model: ex. 27 songs/1 bathroom break

These relationships are called **Conversion Factors**.

What are the components of a conversion factor?

Using complete sentences consult with your group and come up with a description of a conversion factor. What are its essential components and what is its purpose?

11. Which one of the conversion factors from #10 would you use to determine how long it would take to burn 8 gallons of gas?

3 gal/75 min

12. Construct the conversion factor needed to determine how many songs you would hear in 500 miles. 27 songs/90 miles

13. Solve # 12 mathematically. Show your work below and be sure to include units.

$$\frac{90 \text{ mi}}{27 \text{ songs}} = \frac{500 \text{ mi}}{x \text{ songs}} \quad \text{or} \quad \frac{500 \text{ mi}}{1} \times \frac{27 \text{ songs}}{90 \text{ mi}} = 150 \text{ songs}$$

Reflections:

14. As a group, write grammatically correct English sentences to describe the objective of the activity at this point. Be prepared to share your answer with the class.

Various responses

15. After having shared with the class, does your group still agree with your initial assessment of what the objective is?

Various responses

16. As a group, can you think of a situation when a scientist or chemist might need to use conversion factors to solve a problem? Give an example.

Various responses

Exercises:

Using **conversion factors** to solve a problem is called **Dimensional Analysis**. You should now be able to solve the following problems.

17. Solve this problem without using a calculator:

$$\frac{6 \times \cancel{17} \times 3 \times \cancel{13}}{\cancel{13} \times 9 \times \cancel{17}} = 2$$

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18. Write a mathematical rule that makes this problem easier to solve.

Terms common to both numerator and denominator can be cancelled or reduced.

19. Solve this problem:

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$$\frac{\text{miles} \times \text{songs} \times \text{gallons}}{\text{miles} \times \text{gallons}} = \text{songs}$$

20. How many miles would you have to drive to hear 43 songs? Show how you solve the problem using units and conversion factors.

$$\frac{43 \text{ songs}}{1} \times \frac{90 \text{ mi}}{27 \text{ songs}} = 143.3 \text{ mi}$$

21. Using your answer from # 20, how many minutes would this take? Again show how you solve the problem using units and conversion factors.

$$\frac{143.3 \text{ mi}}{1} \times \frac{75 \text{ min}}{90 \text{ mi}} = 119.4 \text{ min}$$

22. Show how you can combine problems # 20 and # 21 into one. Draw a line through any units that cancel. Put your answer on the board.

$$\frac{43 \text{ songs}}{1} \times \frac{90 \text{ mi}}{27 \text{ songs}} \times \frac{75 \text{ min}}{90 \text{ mi}} = 119.4 \text{ min} \quad \text{or} \quad \frac{43 \text{ songs}}{1} \times \frac{75 \text{ min}}{27 \text{ songs}} = 119.4 \text{ min}$$

23. Write a grammatically correct English sentence to describe which unit you will be left with in the answer. **The final answer should be minutes.**

On your own

24. The average human heart beats 72 beats/minute. If you live to be 80 years old, how many times does your heart beat? What conversion factors do you need to know to solve this problem? List these conversion factors.

$$\frac{365 \text{ day}}{1 \text{ year}} \quad \frac{24 \text{ hr}}{1 \text{ day}} \quad \frac{60 \text{ min}}{1 \text{ hr}} \quad \frac{72 \text{ beats}}{1 \text{ min}}$$

25. What should be the units of the answer? What value would you use to begin the problem and why? Solve the problem and show your work. Include all units and show cancellations of the units.

The answer should be beats. 80 years is the starting value. It does not have fractional units and so is not a conversion factor.

$$\frac{80 \text{ yr}}{(1)} \times \frac{365 \text{ day}}{1 \text{ yr}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{72 \text{ beats}}{1 \text{ min}} = \sim 3,000,000,000$$
